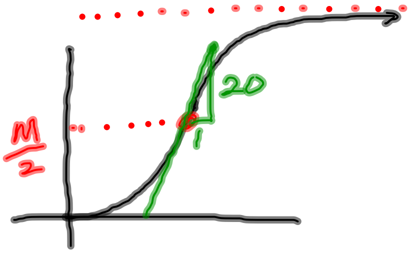
6.5 logistic growth



starts exponential, but it levels off.

M = carrying capacity (max sustainable population)

d.e.

$$\frac{dy}{dt} = ky(M-y)$$

inflection point at $y = \frac{M}{2}$

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Example $\frac{dP}{dt} = .008\,P(100-P)$ bear pop

a) what is the carrying capacity? 100 bears

b) what is the population when ~~it~~ the pop is growing fastest? 50 bears

c) what is the growth rate when the pop. is growing fastest.

$.008 \cdot 50 \cdot (100-50)$

$20 \ \frac{\text{bears}}{\text{year}}$

when there are 50 bears, the pop. will increase by about 20 bears in the next year

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How to solve $\frac{dy}{dt} = .008\, y\,(100-y)$

(find y)

seperate var. $\frac{dy}{y(100-y)} = .008\, dt$

integrate $\int \frac{1}{y(100-y)}\, dy = \int .008\, dt$

partial fractions $\int \frac{A}{y} + \frac{B}{100-y}\, dy = \int .008\, dt$

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long way to find A, B $\frac{1}{y(100-y)} = \frac{(100-y)}{(100-y)}\frac{A}{y} + \frac{B}{(100-y)}\frac{y}{y}$

Common denominator $\frac{0y+1}{y(100-y)} = \frac{100A - Ay + By}{y(100-y)}$

$-Ay + By = 0$

$100A = 1$

$A = \frac{1}{100}$

$y(-A+B) = 0 \qquad B = \frac{1}{100}$

shortcut

cover up method

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$$\int \frac{1/100}{y} + \frac{1/100}{100-y}\, dy = \int .008\, dt$$

$$\frac{1}{100}\int \frac{1}{y} + \frac{1}{100-y}\, dy = \int .008\, dt$$

$$\frac{1}{100}\left(\ln y + -\ln(100-y) \right) + C_1 = .008t + C_2$$

$$e^{\ln\left(\frac{y}{100-y}\right)} = e^{(.8t\ +\ c)}$$

$$\frac{y}{100-y} = e^{.8t+c} = e^{.8t}\cdot e^{c}$$

solve for y

$$\frac{y}{100-y} = c\, e^{.8t}$$

Init. cond. $t=0,\ y=y_0$

$$c = \frac{y_0}{100-y_0}$$

$$\frac{100-y}{\cancel{y}} = \frac{1}{ce^{.8t}}\, y$$

what if $y_0 = 10$? $A = 9$

$$100 = \frac{1}{ce^{.8t}} y + y \quad \text{factor}$$

$A = \frac{1}{c}$

$A = \frac{100-y_0}{y_0}$

$$\frac{100}{1+Ae^{-.8t}} = \cancel{\left(\frac{1}{ce^{.8t}} + 1\right)}\, y$$

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$$\frac{dP}{dt} = kP(M-P)$$

solution $\boxed{P = \frac{M}{1+Ae^{-mkt}}}$

$$\boxed{A = \frac{M-y_0}{y_0}}$$

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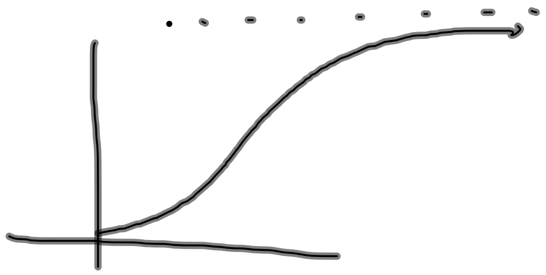
$$\text{deSolve}\left(p' = .0003\,p(1000-p),\ \overset{t}{\cancel{p}},\ \overset{p}{\cancel{t}}\right)$$

Ex 5

$$\frac{dp}{dt} = .0003\,p\,(1000-p)$$

$$A = \frac{1000-61}{61} \qquad p = \frac{1000}{1 + 15.393e^{-.3t}}$$

$$A = 15.393$$



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